

Consider this input:
 $x=23$ and $y=7$

Pre-condition: $23 \geq 0$ & $7 > 0$ & $23, 7$ are \mathbb{N}

1: $q=0$

2: $r=23$

0-th iteration of while loop: $23 = (0 \cdot 7) + 23$ and $23 \geq 0$ - loop invariant holds

3: start a while loop ($7 \leq 23$)

4: $r=23-7=16$

5: $q=0+1=1$

6: end while loop

1-st iteration of while loop: $23 = (1 \cdot 7) + 16$ and $16 \geq 0$ - loop invariant holds

3: run while loop ($7 \leq 16$)

4: $r=16-7=9$

5: $q=1+1=2$

6: end while loop

2-nd iteration of while loop: $23 = (2 \cdot 7) + 9$ and $9 \geq 0$ - loop invariant holds

3: run while loop ($7 \leq 9$)

4: $r=9-7=2$

5: $q=2+1=3$

6: end while loop

3-rd iteration of while loop: $23 = (3 \cdot 7) + 2$ and $2 \geq 0$ - loop invariant

3: run while loop (not $7 \leq 2$)

7: return 3, 2

Post-condition: $23 = (3.7) + 2$ & $0 \leq 2 < 7$

What would happen if $y > x$; say $x = 5$ and $y = 6$

Precondition is met ($x \geq 0$ and $y > 0$ and they are natural numbers)

1: $q = 0$

2: $r = 5$

3: loop does not run since $y = 6 > r = 5$

7: return $q = 0, r = 5$

Postcondition: $5 = (0.6) + 5$ and $0 \leq 5 < 6$

Proving correctness of an algorithm:

1. Come up with pre/post conditions
2. Then a loop invariant to link them
3. Prove loop invariant by induction
4. Use loop invariant to prove partial correctness
5. Prove termination, and with partial correctness this gives (full) correctness

How to compute an estimate of running time (i.e., number of commands executed) for A1.1 for a given x and y .